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Modeling of Gossamer Space Structures with Distributed Transfer Function Method

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A new structural modeling and analysis method, the distributed transfer function method, is presented for application to gossamer space structures. The distributed transfer function method uses distributed transfer functions, instead of shape function used by traditional finite element solvers, to represent the displacement field. The distributed transfer function method maintains the modeling flexibility of the finite element method, so that it is capable of modeling multibody complex structures, but it requires much fewer nodes and results in a significant reduction of computational time. The distributed transfer functions give rise to closed-form analytical solutions of both displacement and strain fields. As a result, the distributed transfer function method only decomposes a structure at those points where multiple components are connected, to keep each component as large as possible. Gossamer space structures are generally composed of several long booms and large membranes. Therefore, the distributed transfer function method can be used to model a gossamer structure with a small number of unknowns and matrices of low order. It offers very accurate results with high computational efficiency. The distributed transfer function method is applied to investigate the sensitivity of buckling strength of an inflatable/rigidizable boom to the variations in bending stiffness.

Nomenclature

a_{ijk}	=	constants (may represent stiffness and other characteristics)
b_{ijk}	=	constants (may represent damping, gyroscopic, and other characteristics)
c_{ijk}	=	constants (may represent inertia and other characteristics)
E	=	modulus of elasticity, psi (Pa)
$\bar{E}$	=	constitutive matrix
$f_i(x,t)$	=	external disturbances, lb, lb·in. (N, N · m)
I	=	moment of inertia, in.^{4} (m^4)
$\bar{I}$	=	identity matrix
L	=	length of a one-dimensional component, in. (m)
L_1	=	length of a boom, in. (m)
M	=	boundary selection matrix
N	=	boundary selection matrix
N_j	=	highest order of differentiation of u_j with respect to x
n	=	number of differential equations, equal to the number of unknown displacement functions $u_i(x,t)$
s	=	complex variable of the Laplace domain
$u_i(x,t)$	=	displacements, in. (m)
w	=	deflection, in. (m)
$\varepsilon_k(x)$	=	strain vector
$\boldsymbol{\eta}(x,s)$	=	state space vector
$\boldsymbol{\sigma}_k(x)$	=	force vector
$\Phi(x)$	=	fundamental matrix
φ	=	stiffness variation

Introduction

GOSSAMER space systems are generally composed of supporting structures formed by highly flexible, long tubular elements and pretensioned thin-film membranes. Figures 1, 2, and 3, respectively, show three examples of gossamer structures: an inflatable sunshield, a solar sail, and an inflatable reflect-array antenna. They all consist of several inflatable booms and single-layer or multiple-layers membrane. Gossamer systems offer order-of-magnitude reductions in mass and launch volume and will revolutionize the architecture and design of space flight systems that require large in-orbit configurations and apertures. Recently, a great deal of interest has been generated in flying gossamer systems on near-term and future space missions.[1]

Modeling, analysis, and optimization are essential to the success of the development and deployment of gossamer structures. However, these tasks are complicated due to a variety of issues such as formation and effects of wrinkles in tensioned membranes, synthesis of tubular and membrane elements into a complete structural system, buckling analysis of inflatable boom components with material and geometric imperfections, and optimization of design with non-uniformly distributed boom structures.

Gossamer structures are usually analyzed by using the finite element method (FEM). FEM is a very powerful numerical method due to its flexibility in modeling multiple complicated geometric bodies with arbitrary boundary conditions.[2] However, this flexibility is costly due to the need of large computer storage and long computational time for solving complex structural problems such as those typical of gossamer structures. FEM is also impractical for real-time computations, such as those necessary for active control of gossamer systems.

The finite strip method is another numerical method that may be used for two-dimensional gossamer components.[3] In this method, a two-dimensional component is discretized into a number of strips. The strip displacements are approximated by polynomials and series of continuous functions. It has been demonstrated

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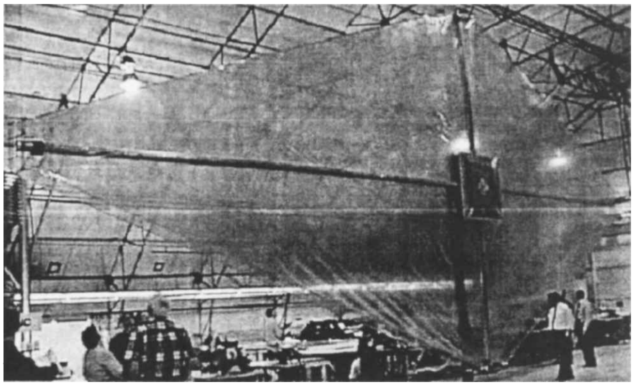

Fig. 1 Inflatable sunshield.

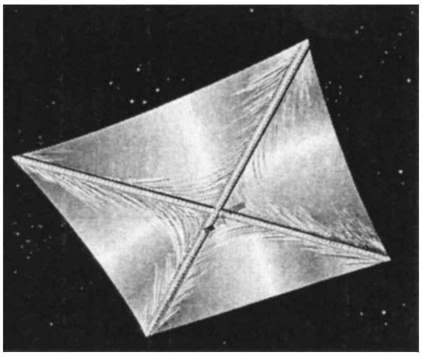

Fig. 2 Solar sail.

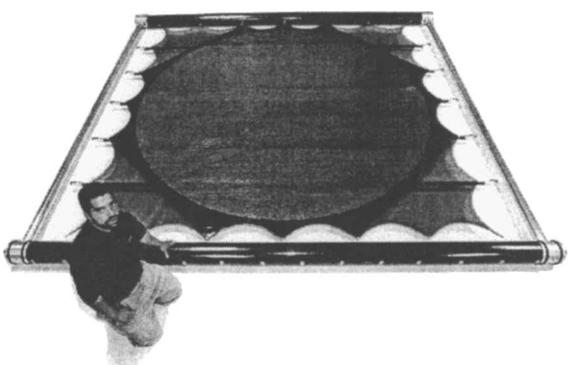

Fig. 3 Inflatable reflect-array antenna.

that the finite strip method requires much less computer memory. Nonetheless, the finite strip method is limited to certain regular shapes. Ritz and Galerkin methods are other numerical methods used for structural analyses.[4] These methods use series of continuous functions that satisfy prescribed boundary conditions as solutions. These admissible/comparison functions are selected on problem-by-problem bases. High accuracy of series solutions relies on the large number of functions in the series. These features prevent series solution methods from being extended to general multibody gossamer systems.

In traditional FEM codes, the element displacements are approximated using known shape functions and unknown nodal displacements, which are the displacements at the selected points on the elements. As a result, a multibody complex system can be systematically assembled from these finite elements by imposing displacement continuity and force equilibrium conditions on the element functions. Because FEM relies on fairly simple shape functions for interpolating the displacement field, a large number of elements are essential to obtain accurate results for problems in which displacements vary dramatically across the structure.

It can be seen from Figs. 1–3 that gossamer structures are generally composed of large membranes that are supported by several long

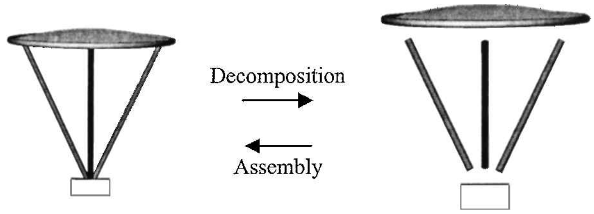

Fig. 4 Decomposing and assembling of a gossamer structure.

booms. Both the membranes and the booms can exhibit local wrinkling and complex variations in their displacement fields. Hence, this study is motivated by the desire to have an analysis methodology for gossamer structures that has the flexibility of FEM in treating multibody systems, but that yields highly accurate solutions for structures with local wrinkles and complex displacement variations. Extending the assembly concept of FEM, if one can find closed-form and analytical or semi-analytical solutions that are expressible by "nodal parameters," then treating every boom and membrane as a "super element," one can construct highly accurate and memory-saving solutions for gossamer structures.

The distributed transfer function method (DTFM) presented by this study for gossamer structures is based on this idea. DTFM uses distributed transfer functions[5] instead of shape functions of FEM to represent the displacement fields. DTFM is distinctively suitable for simulation of gossamer structures. DTFM models a gossamer structure with a minimum number of nodes. This is done by decomposing the structure only at those points where multiple structural components are connected and by keeping each component as large as possible. Figure 4 shows an example. A gossamer structure is usually composed of several very long tubular components, as well as several layers of membranes. Those basic building blocks are connected to each other at a small number of points. As a result, the DTFM models the gossamer structure with a small number of unknowns and deals with low-order matrices. Furthermore, the DTFM gives closed-form analytical or semi-analytical solutions, which renders the DTFM-based analysis results more reliable.

The remainder of the paper is arranged as follows: First, the general process of the DTFM is presented. Then, an example to demonstrate the calculation efficiency of DTFM is given. Finally, the sensitivity analysis procedure with respect to the deviation of the boom bending stiffness is discussed with an example.

DTFM

In a DTFM-based modeling and analysis process, a complex structural system is first decomposed into several structural components (substructures). How a structure is decomposed into substructures and assembled later on is shown in Fig. 4. Unlike the FEM approach, which needs to further divide the substructures into small elements, the DTFM approach treats each substructure as a single component. Therefore, the DTFM approach leads to a much smaller number of unknowns to be determined. Following the decomposition, the governing differential equations of the substructures are then Laplace transformed (with respect to time). Dynamic stiffness matrices, which include the frequency as a variable, can then be established based on distributed transfer function solutions and systematically assembled to form the original structure. Unlike the FEM, the DTFM gives not only displacements, but also higher-order derivatives (strains and stresses) with closed-form solutions that precisely describe the behaviors of the structure. Without loss of generality, one-dimensional components will be used to show the DTFM modeling process in the following paragraphs.

In the local coordinate system, the displacements of a single one-dimensional distributed component are governed by linear partial differential equations:

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j}\left(a_{ijk}+b_{ijk}\frac{\partial}{\partial t}+c_{ijk}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^k u_j(x,t)}{\partial x^k}=f_i(x,t)$$

$$x\in(0,L),\qquad t\geq 0,\qquad i=1,\ldots,n \tag{1}$$

Coefficients a_{ijk}, b_{ijk}, and c_{ijk} represent inertia, damping, and distributed constraint, for example, elastic foundation, gyroscopic term, axial load, etc. Equations (1) represent various one-dimensional continua such as a beam, frame, truss, rotating shaft, etc.

Equations (1) are now Laplace transformed with respect to time t and expressed as

$$\sum_{j=1}^{n}\sum_{k=0}^{N_j} D_{ijk}\frac{\mathrm{d}^k\bar{u}_j(x,s)}{\mathrm{d}x^k} = \bar{f}_i(x,s), \quad x\in(0,L), \quad i=1,\ldots,n \tag{2}$$

with

$$D_{ijk} = \left(a_{ijk} + b_{ijk}s + c_{ijk}s^2\right) \tag{3}$$

Where the overbar denotes the Laplace transformation with the assumptions of zeros for the initial conditions.

Equation (3) is cast into a state-space form:

$$\frac{\mathrm{d}}{\mathrm{d}x}\boldsymbol{\eta}(x,s) = F(s)\boldsymbol{\eta}(x,s) + q(x,s), \qquad x\in(0,L) \tag{4}$$

The boundary conditions of this component are assumed to be given by the equation

$$M\eta(0,s) + N\eta(L,s) = r(s) \tag{5}$$

Entries of two boundary selection matrices, M and N, can be easily changed to assign different boundary conditions.

If the boundary value problem defined by Eqs. (4) and (5) with $q(x,s)=0$ and $r(s)=0$ has only the null solution, then the solution of the state-space vector can be given by the following expression[5]:

$$\boldsymbol{\eta}(x,s) = \int_0^L G(x,\zeta,s)q(\zeta,s)\,\mathrm{d}\zeta + H(x,s)r(s) \tag{6}$$

where

$$G(x,\zeta,s) = \begin{cases} e^{F(s)x}\left(M+Ne^{F(s)L}\right)^{-1}Me^{-F(s)\zeta}, & \zeta\le x \\ -e^{F(s)x}\left(M+Ne^{F(s)L}\right)^{-1}Ne^{F(s)(L-\zeta)}, & \zeta\ge x \end{cases} \tag{7}$$

$$H(x,s) = e^{F(s)x}\left(M+Ne^{F(s)L}\right)^{-1} \tag{8}$$

are distributed transfer functions and $e^{F(s)x}$ is the fundamental matrix of the component.

The state-space vector $\boldsymbol{\eta}(x,s)$ can be divided into two subvectors and expressed as

$$\boldsymbol{\eta}(x,s) = [\boldsymbol{\alpha}^T(x,s) \quad \varepsilon^T(x,s)]^T \tag{9}$$

where

$$\boldsymbol{\alpha}(x,s) = \left[\alpha_1^T(x,s) \quad \alpha_2^T(x,s) \quad \cdots \quad \alpha_n^T(x,s)\right]^T \tag{10}$$

is the displacement vector and

$$\varepsilon(x,s) = \left[\varepsilon_1^T(x,s) \quad \varepsilon_2^T(x,s) \quad \cdots \quad \varepsilon_n^T(x,s)\right]^T \tag{11}$$

is the strain vector.

The force vector $\boldsymbol{\sigma}(x,s)$ at any point along the component can then be calculated and expressed as

$$\boldsymbol{\sigma}(x,s) = \bar{E}\varepsilon(x,s) \tag{12}$$

Correspondingly, force vectors at two ends of the component can be calculated by Eq. (12) and expressed as

$$\begin{bmatrix}\boldsymbol{\sigma}(0,s)\\ \boldsymbol{\sigma}(L,s)\end{bmatrix} = \begin{bmatrix}\bar{E}H_{\sigma 0}(0,s) & \bar{E}H_{\sigma L}(0,s)\\ \bar{E}H_{\sigma 0}(L,s) & \bar{E}H_{\sigma L}(L,s)\end{bmatrix}\begin{bmatrix}\boldsymbol{\alpha}(0,s)\\ \boldsymbol{\alpha}(L,s)\end{bmatrix} + \begin{bmatrix}p(0,s)\\ p(L,s)\end{bmatrix} \tag{13}$$

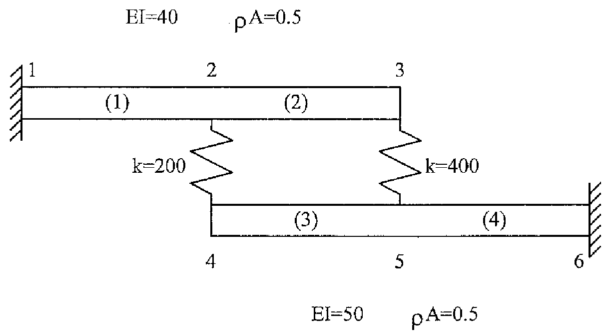

Fig. 5 System with two elastically coupled beams.

In Eq. (13), $\boldsymbol{\sigma}(0,s)$ and $\boldsymbol{\sigma}(L,s)$ are force vectors at two ends of the component, and $\boldsymbol{\alpha}(0,s)$ and $\boldsymbol{\alpha}(L,s)$ are displacement vectors at two ends of the component. Here,

$$k = \begin{bmatrix}\bar{E}H_{\sigma 0}(0,s) & \bar{E}H_{\sigma L}(0,s)\\ \bar{E}H_{\sigma 0}(L,s) & \bar{E}H_{\sigma L}(L,s)\end{bmatrix} \tag{14}$$

is the dynamic stiffness matrix of the component and all of its submatrices can be calculated by using Eq. (8). In Eq. (13) $\boldsymbol{p}(0,s)$ and $\boldsymbol{p}(L,s)$ are force vectors transformed from distributed external forces.

Equation (13) gives the force vectors at two nodes of a component with respect to corresponding displacement vectors. Consequently, dynamic stiffness matrices of all components can be systematically assembled together by using displacement compatibility and force balance at every connecting point to get

$$K(s)\times U(s) = P(s) \tag{15}$$

In Eq. (15), matrix $K(s)$ is the dynamic stiffness matrix of the multicomponents structure, $U(s)$ is the nodal displacement vector of the multicomponents structure, and $P(s)$ is the corresponding nodal force vector. Equation (15) can be used to analyze modal frequencies, mode shapes, frequency responses, time-domain responses, stresses, strains, buckling loads, etc.

Example

Figure 5 shows a system that is composed of two beams elastically coupled by two springs. When DTFM is used, these two beams are decomposed into four components with six nodes, as indicated in Fig. 5. Because of the six given boundary conditions (fixed displacements at nodes 1 and 6, fixed rotations at nodes 1 and 6, and zero force moments on nodes 3 and 4), this system is represented by six unknowns (displacements at nodes 2–5 and rotations at nodes 2 and 5). As a result, DTFM only deals with a six by six matrix to get the closed-form solutions for this system.

Table 1 gives results for the first 12 resonant frequencies of this system calculated by both DTFM and FEM. The second column of Table 1 gives frequencies calculated by DTFM. The third, forth, and fifth columns give frequencies calculated by FEM with 18, 34, and 66 elements, respectively. With more elements, results of FEM converge to DTFM.

Sensitivity Analyses of Gossamer Booms with DTFM

The basic building blocks of most gossamer structures are long booms. The sensitivity of the buckling load of a boom to the cross-sectional deviation is always a challenging problem for engineers.[6] To investigate the sensitivity of the buckling load to the deviation of bending stiffness along a boom, a DTFM-based analysis process has been developed and is presented as follows:

Buckling analysis of a boom can be described by the differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2}{\mathrm{d}x^2}w(x)\right) + P\frac{\mathrm{d}^2}{\mathrm{d}x^2}w(x) = 0 \tag{16}$$

Table 1 Resonant frequencies of the two elastically coupled beam system

Mode	DTFM 6×6 matrix	FEM 18 Elements	34 Elements	66 Elements
1	16.3	16.3	16.3	16.3
2	41.0	41.1	41.0	41.0
3	54.6	53.1	54.2	54.5
4	79.2	77.8	78.9	79.1
5	144.7	138.3	143.1	144.3
6	157.0	150.5	155.4	156.6
7	273.9	258.1	269.9	272.9
8	305.2	288.2	289.9	304.1
9	448.7	415.4	440.4	446.6
10	500.5	463.9	491.2	498.1
11	669.1	601.7	653.7	665.3
12	747.5	672.7	730.5	743.3

Equation (16) belongs to the category defined by Eq. (1). However, because the bending stiffness EI is not a constant along the boom, Eq. (6) cannot be directly employed. The stepwise uniform component was used to handle a nonuniformly distributed component. A nonuniform distributed component is first divided into a number of tiny sections and each section is considered to be uniform. When sections S_k and S_{k+1} are assumed to be interconnected at point x_k, the state-space vector on section S_k at point x_k is given as

$$\boldsymbol{\eta}_k(x) = \begin{bmatrix} u_k(x) \\ \varepsilon_k(x) \end{bmatrix} \tag{17}$$

and the corresponding force vector can be given as

$$\boldsymbol{\sigma}_k(x) = E_k(x)\varepsilon_k(x) \tag{18}$$

Considering the force balance and displacement compatibility at point x_k, the state space vector on section S_{k+1} at point x_k can be calculated as

$$\boldsymbol{\eta}_{k+1}(x_k) = T_k(x)\boldsymbol{\eta}_k(x_k) \tag{19}$$

where

$$T_k = \begin{bmatrix} \bar{I} & 0 \\ 0 & E_{k+1}^{-1}E_k \end{bmatrix} \in C^{n\times n} \tag{20}$$

On the other hand, Eqs. (7) and (8) can be rewritten as

$$G(x,\xi) = \begin{cases} H(x)M\Phi^{-1}(\xi), & \xi \le x \\ -H(x)N\Phi(L)\Phi^{-1}(\xi), & \xi > x \end{cases} \tag{21}$$

$$H(x) = \Phi(x)[M + N\Phi(L)]^{-1} \tag{22}$$

In Eqs. (21) and (22), the fundamental matrix can be approximately expressed as

$$\Phi(x) \approx \hat{\Phi}(x) = \exp[F_{k+1}(x - x_k)]T_k \exp[F_k(x_k - x_{k-1})] \ldots T_2^{F_2(x_2 - x_1)} T_1^{F_1(x_1)}, \qquad x \in (x_k, x_{k+1}) \tag{23}$$

Next an example is given of the buckling sensitivity analysis of a boom to the deviation of bending stiffness. The length of the inflatable/self-rigidizable boom is 197 in. From previous analyses and tests,[7] the original bending stiffness EI_0 of the boom is obtained as 656,673 lb-in.^{2}. To investigate the impact caused by the deviation of bending stiffness, it is assumed that the bending stiffness of the boom is expressed as

$$EI = EI_0[1 + \varphi \times \sin(x\pi/L_1)] \tag{24}$$

Note that the DTFM is able to handle any kind of bending stiffness deviations, even a localized deviation.

The state-space vector for this example is defined as

$$\boldsymbol{\eta}(x) = [w(x) \quad w'(x) \quad w''(x) \quad w'''(x)]^T \tag{25}$$

Based on Eq. (16), the F matrix of Eq. (4) can be derived as

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{P}{EI(x)} & 0 \end{bmatrix} \tag{26}$$

The boundary conditions of the boom are simply supported at both ends. This means that on each end both the deflection and the bending moment are equal to zero:

$$w = 0, \qquad EI\frac{\partial^2 w}{\partial x^2} = 0 \tag{27}$$

which is recast as

$$M\eta(0) + N\eta(L) = 0 \tag{28}$$

with the boundary matrices

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{29}$$

The solution of the beam is given by

$$\boldsymbol{\eta}(x) = \Phi(x; P, \varphi)\boldsymbol{\eta}(0) \tag{30}$$

where $\Phi(x; P, \varphi)$ indicates that the fundamental matrix is a function of the axial load P and the stiffness variation φ. $\Phi(x; P, \varphi)$ can be obtained by the method given in Eq. (23). Substituting Eq. (30) into Eq. (28) gives

$$[M + N\Phi(L; P, \varphi)]\boldsymbol{\eta}(0) = 0 \tag{31}$$

For the preceding homogeneous equation to have a nontrivial solution, we must have

$$\det[M + N\Phi(L; P, \varphi)] = 0 \tag{32}$$

The smallest root of the characteristic equation (32) is the critical force P_{cr} (buckling load) of the nonuniform beam.

Table 2 gives the buckling load as a function of φ. Table 3 gives the ratios of buckling force changing as the function of bending stiffness deviation φ. From Table 3, one can reach the following conclusions:

Table 2 Buckling force (lb) as a function of bending stiffness deviation φ

P_{cr}	φ: 0%	±2%	±4%	±6%	±8%	±10%
+%	167.0	169.7	172.7	175.4	178.2	181.1
−%	167.0	164.2	161.2	158.5	155.6	152.8

Table 3 Ratios of buckling force changing as a function of bending stiffness deviation φ

P_{cr}/P_{cr0}	φ: 0%	±2%	±4%	±6%	±8%	±10%
+%	1.0000	1.017	1.034	1.051	1.067	1.085
−%	1.0000	0.983	0.966	0.949	0.932	0.915

1) Buckling force is almost a linear function of the bending stiffness deviation φ.

2) The percentage changing of buckling force is less than the percentage of bending stiffness deviation φ. For example, buckling force changes 8.5% while φ changes 10%.

Conclusions

The DTFM and two applied examples have been presented. This method is specifically suitable for gossamer structures because gossamer structures are usually composed of only several very long booms and large membranes. The DTFM only decomposes a structure at those points where multiple components are connected, which keeps each component as large as possible. As a result, the DTFM models a gossamer structure with a small number of unknowns and deals with low-order matrices. It gives closed-form analytical solutions with very accurate results and high computational efficiency. The DTFM is also successfully used in this study to investigate the sensitivity of buckling strength of an inflatable/rigidizable boom to a nonuniform (along the length) variation in bending stiffness.

Although not demonstrated herein, the DTFM is convenient for handling structural systems with passive and active damping, gyroscopic effects, embedded smart material layers as sensing and actuating devices, and feedback control.

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References

[1] Lou, M., "Development and Application of Space Inflatable Structures," *Proceedings of the 22nd International Symposium on Space Technology and Science*, Japan Society for Aeronautical and Space Science, 2000.

[2] Cook, R. D., Malkus, D. S., and Plesha, M. E., *Concepts and Applications of Finite Element Analysis*, Wiley, New York, 1989.

[3] Cheung, Y. K., *Finite Strip Method in Structural Analysis*, Pergamon, Oxford, 1976.

[4] Meirovitch, L., *Analytical Methods in Vibrations*, Macmillan, New York, 1967.

[5] Yang, B., and Tan, C. A., "Transfer Functions of One-Dimensional Distributed Parameter Systems," *Journal of Applied Mechanics*, Vol. 59, 1992, pp. 1009–1014.

[6] Lake, M. S., and Georgiadis, N., "Analysis and Testing of Axial Compression in Imperfect Slender Truss Struts," NASA TM-904174, 1990.

[7] Lou, M., Fang, H., and Hsia, L. M., "Development of Space Inflatable/Rigidizable STR Aluminum Laminate Booms," AIAA Paper 2000-5296, Sept. 2000.

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